

## PERT/CPM

**PERT – Program Evaluation & Review Technique** – It is generally used for those projects where time required to complete various activities are not known as *a priori*. It is probabilistic model & is primarily concerned for evaluation of time. It is event oriented.

**CPM – Critical Path Analysis** – It is a commonly used for those projects which are repetitive in nature & where one has prior experience of handling similar projects. It is a deterministic model & places emphasis on time & cost for activities of a project.

- A project can be defined as a set of large number of activities or jobs (with each activity consuming time & resources) that are performed in a certain sequence determined.
- A network is a graphical representation of a project, depicting the flow as well as the sequence of well-defined activities & events.
- An activity (**Also known as task & job**) is any portion of a project which consumes time or resources and has definable beginning & ending.
- Event (**Also known as node & connector**) is the beginning & ending points of an activity or a group of activities.

### Steps for drawing CPM/PERT network:

1. Analyze & break up of the entire project into smaller systems i.e. specific activities and/or events.
2. Determine the interdependence & sequence of those activities.
3. Draw a network diagram.
4. Estimate the completion time, cost, etc. for each activity.
5. Identify the critical path (longest path through the network).
6. Update the CPM/PERT diagram as the project progresses.

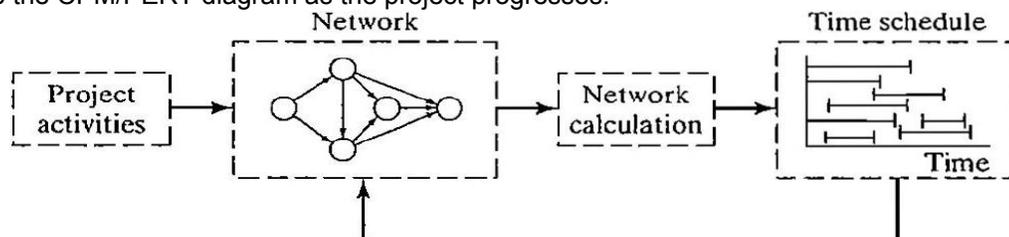


FIGURE: Phases for project planning with CPM-PERT

### Network Representation:

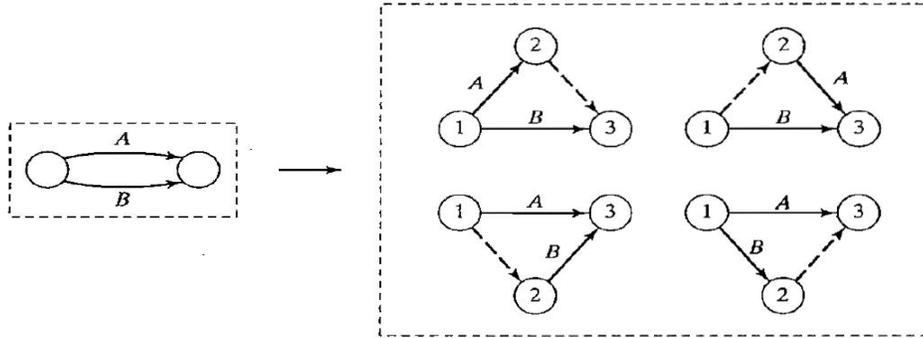
Each activity of the project is represented by arrow pointing in direction of progress of project. The events of the network establish the precedence relationship among different activities. Three rules are available for constructing the network.

**Rule 1.** Each activity is represented by one & only one, arrow.

**Rule 2.** Each activity must be identified by two distinct events & No two or more activities can have the same tail and head events.

Following figure shows how a dummy activity can be used to represent two concurrent activities, A & B. By definition, a dummy activity, which normally is depicted by a dashed arrow, consumes no time or resources. Dummy activity is a hypothetical activity which takes no resource or time to complete. It is represented by broken arrowed line & is used for either distinguishing activities having common starting & finishing events or to identify & maintain proper precedence relationship between activities that are not connected by events.

FIGURE Use of dummy activity to produce unique representation of concurrent activities



Inserting dummy activity in one four ways in the figure, we maintain the concurrence of A & B, and provide unique end events for the two activities (to satisfy Rule 2).

**Rule 3.** To maintain correct precedence relationship, the following questions must be answered as each activity is added to the network:

- (a) What activities must be immediately precede the current activity?
- (b) What activities must follow the current activity?
- (c) What activities must occur concurrently with the current activity?

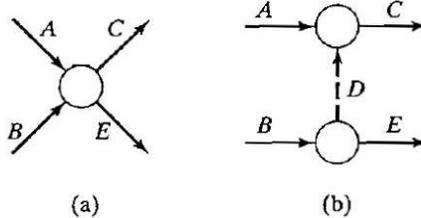


FIGURE Use of dummy activity to ensure correct precedence relationship

The answers to these questions may require the use of dummy activities to ensure correct precedences among the activities. For example, consider the following segment of a project:

1. Activity C starts immediately after A and B have been completed.
2. Activity E starts only after B has been completed.

Part (a) of the figure above, shows the incorrect representation of the precedence relationship because it requires both A & B to be completed before E can start. In part (b) the use of dummy rectifies situation.

**Question 1:** What do you mean by a dummy activity? Why it is used in networking? (4 Marks) May/08

**Question 2:** Depict the following dependency relationships by means of network diagrams.(The Alphabets stands for activities)

1. A and B control F; B and C control G.
2. A and B control F; B controls G while C controls G and H.
3. A controls F and G; B controls G while C controls G and H.
4. A controls F and G; B and C control G with H depending upon C.
5. F and G are controlled by A, G and H are controlled by B with H controlled by B and C.
6. A controls F, G and H; B controls G and H with H controlled by C.

**Question 3:** Develop a network based on the following information;

Activity	Immediate predecessors
A	-
B	-
C	A
D	B
E	C,D
F	D
G	E

H

F

**Question 4:** Construct the project network comprised of activities A to L with the following precedence relationships:

- (a) A, B and C, the first activities of the project can be executed concurrently
- (b) A & B precede D
- (c) B precedes E, F, H
- (d) F and C precede G
- (e) E and H precede I & J
- (f) C, D, F and J precede K
- (g) K precede L
- (h) I, G, and L are terminal activities of the project.

**Question 5:** Construct the project network comprised of activities A to P that satisfies the following precedence relationships:

- (a) A, B and C, the first activities of the project can be executed concurrently
- (b) D, E and F follow A
- (c) I and G follow both B and D
- (d) H follows both C & G
- (e) K and L follow I
- (f) J succeeds both E and H
- (g) M and N succeed F, but cannot start until both E and H are completed.
- (h) O succeeds both M and I
- (i) P succeeds J, L and O
- (j) K, N and P are the terminal activities of the project.

**Question 6:** A publisher has a contract with an author to publish a textbook. The simplified (activities) associated with the production of the textbook are given below. The author is required to submit to the publisher a hard copy and a computer file of the manuscript. Develop the associated network for the project.

Activity	Predecessor(s)	Duration (weeks)
A: Manuscript proofreading by editor	—	3
B: Sample pages preparation	—	2
C: Book cover design	—	4
D: Artwork preparation	—	3
E: Author's approval of edited manuscript and sample pages	A, B	2
F: Book formatting	E	4
G: Author's review of formatted pages	F	2
H: Author's review of artwork	D	1
I: Production of printing plates	G, H	2
J: Book production and binding	C, I	4

**Question 7:** A project consists of a series of tasks labeled A, B, ..., H, I with the following relationships (W<X, Y means X and Y cannot start until W is completed; X, Y<W means W cannot start until both X and Y are completed). With this notation construct the network diagram having the following constraints:

$$A < D, E; B, D < F; C < G; B < H; F, G < I$$

**Question 8 (Concurrent Activities):** The footing of a building can be completed in four consecutive sections. The activities for each section include (1) digging, (2) placing steel, and (3) pouring concrete. The digging of one section cannot start until that of the preceding section has been completed. The same restriction applies placing steel & pouring concrete. Develop the project network.

Numbering the Events (Fulkerson's Rule)

1. The initial event which has all outgoing arrows with no incoming arrow is numbered "1".
2. Delete all the arrows coming out from node "1". This will convert some more nodes into initial events. Number these events as 2, 3, 4, ....
3. Delete all the arrows going out from these numbered events to create more initial events. Assign the next numbers to these events.
4. Continue until the final or terminal node, which has all arrows coming in with no arrow going out is numbered.

Determination of time to complete each activity:

The CPM system of networks omits the probabilistic consideration and is based on a Single Time Estimate of the average time required to execute the activity.

In PERT analysis, there is always a great deal of uncertainty associated with the activity durations of any project. Therefore,  $t_e$  estimated time is better described by a probability distribution than by a single estimate. Three time estimates (*from beta probability distribution*) are made as follows:

- 1) The Optimistic Time Estimate ( $t_o$ ): Shortest possible time in which an activity can be completed in ideal conditions. No provisions are made for delays or setbacks while estimating this time.
- 2) The Most Likely Time ( $t_m$ ): It assumes that things go in normal way with few setbacks.
- 3) The Pessimistic Time ( $t_p$ ): The max. possible time if everything go wrong & abnormal situations prevailed. However, major catastrophes such as earthquakes, labour troubles, etc. are not taken into account.

The expected time (mean time) for each activity can be approximated using the weighted average i.e.

$$\text{Expected Time } (t_e) = (t_o + 4t_m + t_p)/6$$

Forward Pass Computation: It is the process of tracing the network from START to END. It gives the earliest start & finish times for each activity.

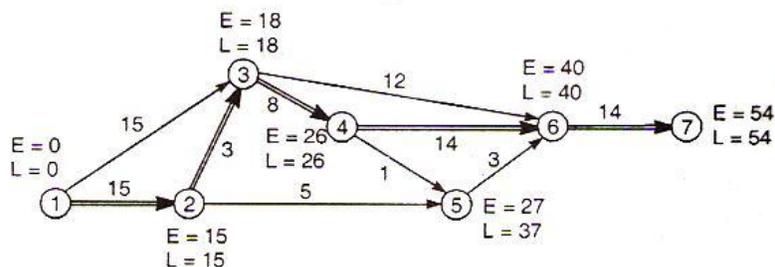
Earliest event time ( $E_j$ ): The time that event  $j$  will occur if the preceding activities are started as early as possible.  $E_j$  is the **max.** of the **sums**  $E_i + t_{ij}$  involving each immediately precedent event  $i$  & intervening event  $ij$ .

Backward Pass Computation: It is the process of tracing the network starting from LAST node & moving backward.

Latest event time ( $L_j$ ): The latest time that event  $i$  can occur without delaying completion of beyond its earliest time.  $L_i$  is the **min.** of the **differences**  $L_i - t_{ij}$  involving each immediately precedent event  $j$  & intervening event  $ij$ .

*E.g.* of Earliest event time & Latest event time:

Activity:	1-2	1-3	2-3	2-5	3-4	3-6	4-5	4-6	5-6	6-7
Duration:	15	15	3	5	8	12	1	14	3	14
(Weeks)										



- In accordance with Zero Slack Convention, if no schedule date for completion of the project is specified, then we take  $L=E$  for the terminal event of the project.
- It is a convention to keep the earliest allowance time of the START event as zero.
- Flexibility of non critical in case of event is known as slack & in case of activity is term as float. (Though some writers have used these terms interchangeably).

The critical path can be identified by determining the following four parameters for each *activity*:

- **EST** - earliest start time: the earliest time at which the activity can start given that all its precedent activities must be completed first =  $E_i$
- **EFT** - earliest finish time, equal to the earliest start time for the activity plus the time required to complete the activity =  $EST(i-j) + t_{ij}$
- **LFT** - latest finish time: the latest time at which the activity can be completed without delaying (beyond its targeted completion time) the project =  $L_j$
- **LST** - latest start time, equal to the latest finish time minus the time required to complete the activity =  $LFT(i-j) - t_{ij}$

**CRITICAL PATH:** The **critical path** is the path through the project network in which none of the activities have float (total float is zero) i.e. A critical path satisfies following 3 conditions:

- $EST = LST$
- $EFT = LFT$
- $E_j - E_i = L_j - L_i = t_{ij}$

The duration of project is fixed by the time taken to complete the path through the network with the greatest total duration. This path is known as critical path & activities on it are known as critical activities. A delay in the critical path delays the project. Similarly, to accelerate the project it is necessary to reduce the total time required for the activities in the critical path.

Dummy Activity may or may not lie on critical path.

**Question 9 (Critical Path):** Tasks A, B, C,....., H, I constitute a project. The precedence relationships are  $A < D; A < E; B < F; D < F; C < G; C < H; F < I; G < I$

Draw a network to represent the project and find the minimum time of completion of the project when time, in days, of each task is as follows:

Task:	A	B	C	D	E	F	G	H	I
Time:	8	10	8	10	16	17	18	14	9

Also identify the critical path.

[Ans.: Critical path is 1-2-4-5-6 with 44 days]

**Question 10:** A project consists of seven activities for which relevant data are given below:

(i) Draw the network (5 Marks)

(ii) Name and highlight the critical path.

Activity	Preceding activity	Activity duration (days)
A	□	4
B	□	7
C	□	6
D	A, B	5
E	A, B	7
F	C, D, E	6
G	C, D, E	5 (1Mark)Nov2006

[Ans.: B, E, F = 20 days]

**Question 11 (Forward and Backward Pass):** A project schedule has the following characteristics:

Activity	Time (weeks)	Activity	Times (week)
1-2	4	5-6	4

1-3	1	5-7	8
2-4	1	6-8	1
3-4	1	7-8	2
3-5	6	8-9	1
4-9	5	8-10	8
		9-10	7

- (i) Construct the PERT network
- (ii) Compute E and L for each event;
- (iii) *Float for each activity*; and
- (iii) Find critical path and its duration.

(10 Marks) May/2000

[Note: Float is to be calculated only after going through below text]

[Ans.: Critical path is 1-3-5-7-8-10 with 25 weeks]

The **total float time** for an *activity* is the time between its earliest and latest start time, or between its earliest and latest finish time. It is the amount of time that an activity can be delayed past its earliest start or earliest finish without delaying the project. = **LST-EST or LFT-EFT** =  $LFT - EST - t_{ij} = LFT - (EST + t_{ij})$

The slack time or **slack of an event** in a network is the difference the latest event time & earliest event time i.e.  $L_i - E_i$

The **free float time** of an *activity* is equal to the amount by which its duration can be increased without affecting either the project time or the time available for the subsequent activities. It indicates the value by which an activity can be delayed beyond the earliest starting point without affecting the earliest start, & therefore, the total float of the activities following it. = **Total Float<sub>ij</sub> – (Slack of event j)**

The **independent float time** of an *activity* is the amount by which the duration of an activity could be extended without affecting the total project time, the time available for subsequent activities or the time available for the preceding activities. = **[Free Float<sub>ij</sub> – (Slack of event i)] or ZERO, whichever is higher**. Also  $EST$  of following activity –  $LFT$  of preceding activity – Duration of current activity or Zero, whichever is higher.

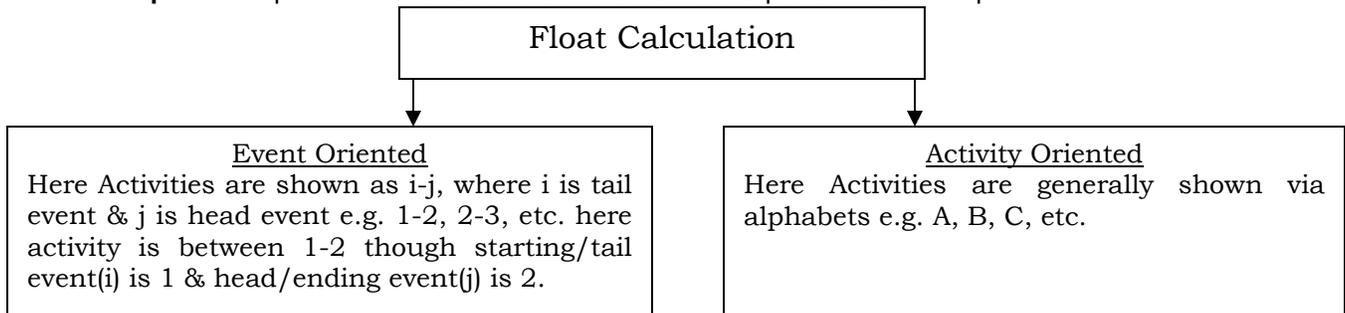
The **interfering float time** is the part of total float which causes a reduction in the float of successor activities. It is that portion of the activity float which cannot be consumed without affecting adversely the float of the subsequent activity or activities. = **LFT – (EST of following activity) or ZERO, whichever is higher**.

While calculating floats, for just for our simplifying computations, we can write values of Slack of event j in column wherein we are supposed to write interfering float.

**Subcritical Activity:** Activity having next higher float than the critical activity.

**Supercritical Activity:** These Activities have negative float. It results when activity duration is more than time available. It indicates abnormal situation requiring as to how to compress the activity.

**Subcritical path:** The path with the next least floats than critical path is subcritical path.

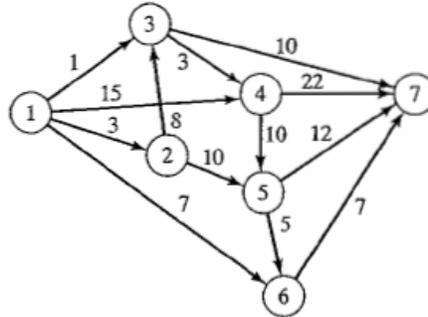


**Question 12 (Floats):** The utility data for a network are given below. Determine the total, free, independent and interfering floats and identify the critical path.

Activity:	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration:	2	8	10	6	3	3	7	5	2	8

**[Ans.: Critical Path is 0-1-3-6-7 with 27]**

**Question 13:** For the network given below, compute E and L for each event & determine the total, free, independent and interfering floats and identify the critical path.



**Question 14:** The following table gives the activities in a construction project and the time duration of each activity:

Activity	Preceding activity	Normal Time (Days)
A	-	16
B	-	20
C	A	8
D	A	10
E	B, C	6
F	D, E	12

Required:

- (i) Draw the activity network of the project.
- (ii) Find critical path.
- (iii) Find the total float and free-float for each activity.

(6 Marks) Nov/07

**[Ans.: (ii) A-C-E-F = 42 days. (iii) Total Float A-0, B-4, C-0, D-4, E-0, F-0; Free Float A-0, B-4, C-0, D-4, E-0, F-0]**

**Question 15:** Given is the following information regarding a project:

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Dependence	-	-	-	AB	B	B	FC	B	EH	EH	CDFJ	K
Duration (days)	3	4	2	5	1	3	6	4	4	2	1	5

Draw the Network Diagram and identify the Critical Path and Project Duration.

(Nov/94)

Find the three types of float (viz. Total, Free and Independent) for each activity.

**[Ans.: B-H-J-K-L = 16 days]**

**Probability Estimate:** It is used to calculate the probability of completing the time within given duration (*Using Normal Distribution*):

$$Z = (T_1 - T_{cp})/\sigma_t$$

Where, Z = Standard Normal Variate

$T_1$  = Duration in which we wish to complete the project

$T_{cp}$  = Duration on critical path

$\sigma_t$  = Standard Deviation of the earliest finish of network = Square root of **sum of** variance of all activity durations of critical path, where

$$\text{Variance Distribution } (\sigma_t^2) = [(t_p - t_o)/6]^2$$

- In case there are two critical paths, variance of separate activities of both of them shall be added for calculating  $\sigma_t$ , but for calculating Z, we will take higher of two  $\sigma_t$  taken above.
- In case of event variance, if there are two longest paths, higher of the two would be picked up.

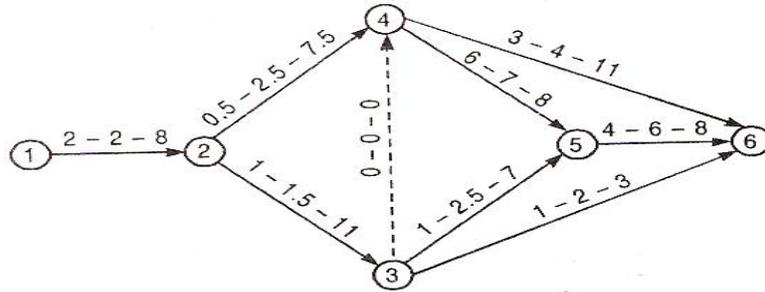
**Question 16:** If the critical path of a project is 20 months along with a standard deviation of 4 months, what is the probability that the project will be completed within: (a) 20 months (b) 18 months (c) 24 months?

[Ans.: 0.50, 0.31, 0.84]

**Question 17:** PERT calculation yield a project length of 60 weeks with variance of 9. Within how many weeks would you expect the project to be completed with probability of 0.99? (That is the project length that you would expect to be exceeded only by 1% of time if the project were repeated many time in an identical manner).

[Ans.: 67 weeks]

**Question 18:** Consider the network shown below. The three time estimates for the activities are given along the arrows. Determine the critical path. What is the probability that the project will be completed in 20 days?



[Ans.: 0.6844]

**Question 19:** Consider the schedule of activities and related information as given below, for the construction of a plant:

Activity	Expected Time (Months)	Variance	Expected Cost (Millions of Rs.)
1-2	4	1	5
2-3	2	1	3
3-6	3	1	4
2-4	6	2	9
1-5	2	1	2
5-6	5	1	12
4-6	9	5	20
5-7	7	8	7
7-8	10	16	14
6-8	1	1	4

Assuming that the cost and time required for one activity is independent of the time and cost of any other activity are expected to follow normal distribution.

Draw a network based on the above data and calculate:

- Critical path
- Expected cost of construction of the plant.
- Expected time required to build the plant.
- The standard deviation of the expected time.

(10 Marks) May/01

[Ans.: (a) 1-2-4-6-8; (b) Rs. 80 millions; (c) 20 months; (d) 3 months]

**Question 20:** A project consists of seven activities and the time estimates of the activities are furnished as under:

Activity	Optimistic Days	Most likely Days	Pessimistic Days
1-2	4	10	16
1-3	3	6	9
1-4	4	7	16
2-5	5	5	5
3-5	8	11	32
4-6	4	10	16
5-6	2	5	8

Required:

- (i) Draw the network diagram.
  - (ii) Identify the critical path and its duration.
  - (iii) What is the probability that project will be completed in 5 days earlier than the critical path duration?
  - (iv) What project duration will provide 95% confidence level of completion ( $Z_{0.95} = 1.65$ )?
- Given (11 Marks) Nov/08-Old Course

Z	1.00	1.09	1.18	1.25	1.33
Probability	0.1587	0.1379	0.1190	0.1056	0.0918

[Ans.: (ii) Critical Path is 1→3→5→6 & its duration is 25 days (iii) Probability = 11.90%, (iv) 32 days (approx)]

**Question 21:** The time estimate (in weeks) for the activities of a PERT network are given below:

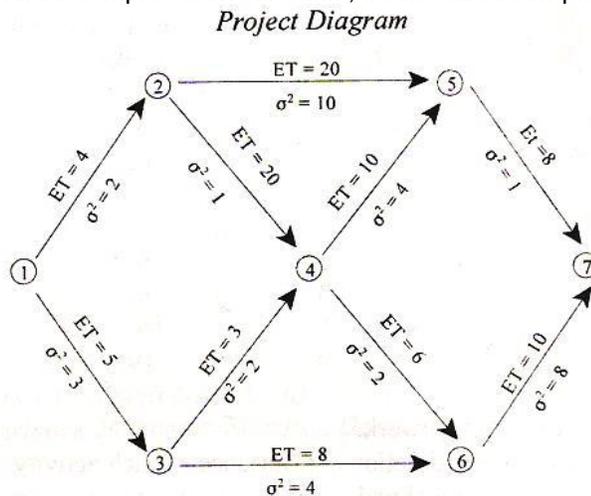
Activity	$t_o$	$t_m$	$t_p$
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

- (a) Draw the project network and identify all the paths through it.
- (b) Determine the expected project length.
- (c) Calculate the standard deviation and variance of the project length.
- (d) What is the probability that the project will be completed.
  - 1. at least 4 weeks earlier than expected time?
  - 2. no more that 4 weeks later than expected time?
- (e) If the project due date is 19 weeks, what is the probability of not meeting the due date?
- (f) The probability that the project will be completed on schedule if the scheduled completion time is 20 weeks.
- (g) What should be the scheduled completion time for the probability of completion to be 90 %? (Nov.1991)

[Ans.: (c) Variance = 9 & Standard Deviation = 3; (d) (i) 0.0918 (ii) 0.9082 (e) 0.2514 (f) 0.8413 (g) 20.84]

**Question 22:** Given the following project network, determine:

- 1. Earliest expected completion time for each event
- 2. Latest allowable completion time for each event
- 3. Slack time for each event
- 4. Critical Path
- 5. The probability that project will be completed on schedule, if scheduled completion time is 38



(8 Marks) Nov./04

[Hint: Critical Path is 1-2-4-5-7 and probability = 7.93%]

**Question 23:** A small project is composed of seven activities, whose time estimates are listed below. Activities are identified by their beginning (i) and (j) node number.

Activity (i-j)	Estimated durations (in days)		
	Optimistic	Most Likely	Pessimistic
1-2	2	2	14
1-3	2	8	14
1-4	4	4	16
2-5	2	2	2
3-5	4	10	28
4-6	4	10	16
5-6	6	12	30

- Draw the project network.
- Find the expected duration and variance for each activity. What is the expected project length.
- What is the probability that project will be completed at least 8 days earlier than expected?
- If the project due date is 38 days, what is the probability of not meeting the due date?

Given: z	0.50	0.67	1.00	1.33	2.00
P	0.3085	0.2514	0.1587	0.0918	0.0228

[Hint: The expected duration of the project = 34 days; Probability of meeting the due date is 9.18%; Probability not meeting the due date is 25.14%] (7 Marks) Nov/05

**Question 24:** A project consists of the following activities, whose time estimates are given against each as under:

Activity	Estimated duration (weeks)		
	Optimistic	Most likely	Pessimistic
1-2	3	6	15
1-3	2	5	14
1-4	6	12	30
2-5	2	5	8
2-6	5	11	17
3-6	3	6	15
4-7	3	9	27
5-7	1	4	7
6-7	4	19	28

Required :

- Draw the project net work.
- Find the expected duration and variance of each activity.
- Determine the critical path and the expected project duration.
- What is the probability that the project will be completed in 38 weeks?
- What project duration will have 95% chance of completion. ( $Z_{0.95} = 1.65$ )

Given : Z	0.21	0.41	0.82
	0.0832	0.1591	0.2939

(8 Marks) May/03

[Ans.: (iii) Critical path 1→2→6→7, Expected project duration is 36 weeks. (iv) 66% (v) 44 weeks]

**Question 25:** An Engineering Project has the following activities, whose time estimates are listed below:

Activity (i-j)	Estimated Duration (in months)		
	Optimistic	Most Likely	Pessimistic
1-2	2	2	14
1-3	2	8	14
1-4	4	4	16
2-5	2	2	2
3-5	4	10	28
4-6	4	10	16

5-6	6	12	30
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- (a) Draw the project network and find the critical path.
- (b) Find the expected duration and variance for each activity. What is the expected project length?
- (c) Calculate the variance and standard deviation of the project length.
- (d) What is the probability that the project will be completed at least eight months earlier than expected time?
- (e) If the project due date is 38 months, what is the probability of not meeting the due date? Given:

Given: z	0.50	0.67	1.00	1.33	2.00
P	0.3085	0.2514	0.1587	0.0918	0.0228

[Ans.: (i) Critical path 1-3-5-6; (ii) Expected project length 34 months; (iii) Variance 36 months, Standard Deviation 6; (iv) 9.18%; (v) 25.14%]  
(10 Marks) Nov./01

**Question 26:** A civil engineering firm has to bid for the construction of a dam. The activities and their time estimates are given below:

Activity	Optimistic	Most likely	Pessimistic
1-2	14	17	25
2-3	14	18	21
2-4	13	15	18
2-8	16	19	28
3-4 (dummy)	0	0	0
3-5	15	18	27
4-6	13	17	21
5-7 (dummy)	0	0	0
5-9	14	18	20
6-7 (dummy)	0	0	0
6-8 (dummy)	0	0	0
7-9	16	20	41
8-9	14	16	22

The policy of the firm with respect to submitting bids is to bid the minimum amount that will provide a 95% of probability of at best breaking-even. The fixed costs for the project are eight lakhs and the variable costs are 9000 every day spent working on the project. The duration is in days and the costs are in rupees.

What amount should the firm bid under this policy? (You may perform the calculations on duration etc., up to two decimal places)  
[May 1990]

[Ans.: Rs. 1574000]

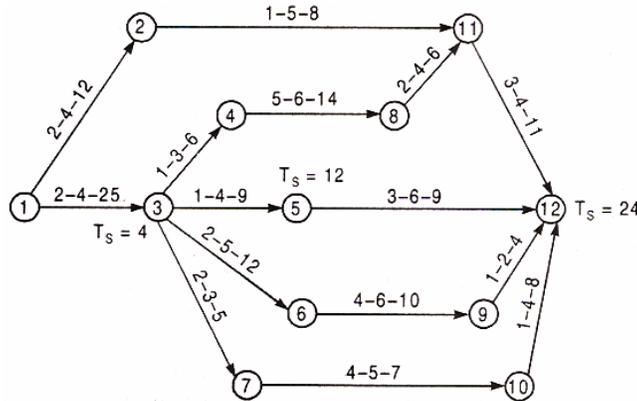
**Question 27:** The optimistic, most likely and pessimistic times of the activities of a project are given below. Activity 40-50 must not start before 22 days, while activity 70-90 must end by 35 days. The scheduled completion time of the project is 46 days. Draw the network and determine the critical path. What is the probability of completing the project in scheduled time?

Activity	$t_o-t_m-t_p$	Activity	$t_o-t_m-t_p$
10-20	4-8-12	50-70	3-6-9
20-30	1-4-7	50-80	4-6-8
20-40	8-12-16	60-100	4-6-8
30-50	3-5-7	70-90	4-8-12
40-50	0-0-0	80-90	2-5-8
40-60	3-6-9	90-100	4-10-16

[Ans.: 10-20-40-50-70-90-100 = 46 days; Probability is 50%]

**Question 28[Calculation for this question is lengthy but probability of non-critical event is a good point]:**

A PERT network is shown below. The activity times in days are given along with the arrows. The scheduled times for some important events are given along the nodes. Determine the critical path and probabilities of meeting the scheduled dates for the specified events. Tabulate the results and determine the slack and for each event.



[Ans.: Critical Path is 1-3-4-8-11-12 with project completion time as 26.51 days, Probability of completing the project in the scheduled completion time of 24 days is 29.54%, Probability that event 3 will occur on scheduled date is 20.44% & Probability of meeting schedule date of event 5 is less than or equal to 54.89% with minimum of 1.8%]

**Question 29:** Distinction between PERT and CPM

(5 Marks) Nov/98 & (5 Marks) Nov/07

PERT	CPM
1. PERT is used for non-repetitive jobs like planning the assembly of the space. 2. it is a probabilistic model. 3. It is event-oriented as the results of analysis are expressed in terms of events or distinct points in time indicative of progress. 4. It is applied mainly for planning and scheduling research programmes. 5. PERT incorporates statistical analysis and thereby determines the probabilities concerning the time by which each activity or entire project would be completed. 6. PERT serves as useful control device as it assists management in controlling a project by calling attention to such delays	1. CPM is used for repetitive job like building a house 2. It is a deterministic model. 3. It is activity-oriented as the result or calculations are considered in terms of activities or operations of the project. 4. It is applied mainly for construction and business problems. 5. CPM does not incorporate statistical analysis in determining time estimates, because time is precise and known. 6. It is difficult to use CPM as a control device for the simple reason that one must repeat the entire evaluation of the project each time the changes are introduced into the network.

**Project Crashing:** There are usually compelling reasons to complete the project earlier than the originally estimated duration of critical path computed on the normal basis of a new project.

**Direct Cost:** This is the cost of the materials, equipment and labour required to perform the activity. When the time duration is reduced the project direct cost increases.

$$\text{Activity Cost Slope} = (C_c - N_c) / (N_t - C_t)$$

Where,  $C_c$  = Crash Cost = Direct cost that is anticipated in completing an activity within crash time.

$N_c$  = Normal Cost = This is the lowest possible direct cost required to complete an activity

$N_t$  = Normal Time = Min. time required to complete an activity at normal cost.

$C_t$  = Crash Time = Min. time required to complete an activity.

**Indirect Cost:** It consists of two parts: fixed cost and variable cost. The fixed cost is due to general and administrative expenses, insurance, etc. Variable indirect cost consists of supervision, interest on capital, etc.

The total project cost is the sum of the direct & the indirect costs.

**Optimum duration** is the project duration at which *total* project cost is lowest.

**Question 30:** A project is composed of seven activities as per the details given below:

Activity	Normal Time (Days)	Crash Time (Days)	Normal Cost (Rs.)	Crash Cost (Rs.)
1-2	4	3	1500	2000

1-3	2	2	1000	1000
1-4	5	4	1875	2250
2-3	7	5	1000	1500
2-5	7	6	2000	2500
3-5	2	1	1250	1625
4-5	5	4	1500	2125

Indirect cost per days of the project is Rs. 500.

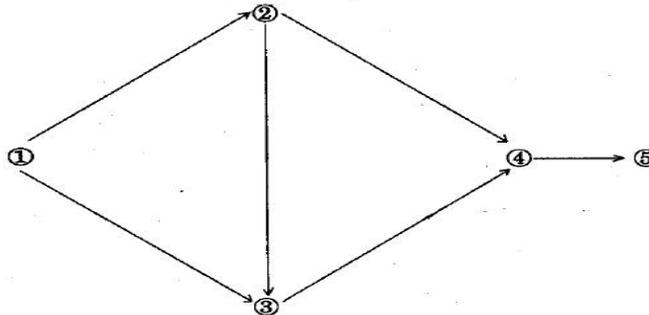
Required:

- Draw the project network.
- Determine the critical path and its duration.
- Find the optimum duration and the resultant cost of the project.

(8 Marks) May/04

[Ans.: (b) Longest path is 13 days; (c) Optimal project duration of 10 days.]

**Question 31:** The normal time, crash time and crashing cost per day are given for the following network:



Activity	Normal Time (Days)	Crash Time (Days)	Crashing Cost (Rs./day)
1-2	18	14	40
1-3	23	22	20
2-3	8	5	60
2-4	10	6	40
3-4	3	2	80
4-5	8	6	50

(i) Crash the project duration in steps and arrive at the minimum duration. What will be the critical path and the cost of crashing?

(ii) If there is an indirect cost of Rs. 70 per day, what will be the optimal project duration and the cost of crashing?

[Ans.: (i) Min. Duration is 30 days & total cost of crashing is Rs. 360 (ii) Optimal Duration is 31 days & cost of crashing is Rs. 280]

(10 Marks) Nov/08-New Course

**Question 32:** A small project consists of jobs as given in the table below. Each job is listed with its normal time and a minimum or crash time (in days). The cost (in Rs. per day) for each job is also given:

Job (i-j)	Normal Duration (in days)	Minimum (crash) Duration (in days)	Cost of Crashing (Rs. per day)
1-2	9	6	20
1-3	8	5	25
1-4	15	10	30
2-4	5	3	10
3-4	10	6	15
4-5	2	1	40

(i) What is the normal project length and the minimum project length?

(ii) Determine the minimum crashing cost of schedules ranging from normal length down to, and including, the minimum length schedule. That is, if L = Length of the schedule, find the costs of schedules which are L, L-1, L-2 and so on.

(iii) Overhead costs total Rs. 60 per day. What is the optimum length schedule in terms of both crashing and overhead cost? List the schedule duration of each job for your solution.

(10 Marks) May/02

[Ans.: (i) Critical path is 1→3→4→5; (ii) Rs. 1155; (iii) Optimum duration of the project is 15 days.]

**Question 33:** The following table gives data on normal time and cost and crash time and cost for a project.

- (d) Draw the network and identify the critical path.  
 (e) What is the normal project duration and associated cost?  
 (f) Find out total float for each activity.  
 (g) Crash the relevant activities systematically and determine the optimum project time and cost.

Activity	Normal		Crash	
	Time (Week)	Cost (Rs.)	Time (Week)	Cost (Rs.)
1-2	3	300	2	400
2-3	3	30	3	30
2-4	7	420	5	580
2-5	9	720	7	810
3-5	5	250	4	300
4-5	0	0	0	0
5-6	6	320	4	410
6-7	4	400	3	470
6-8	13	780	10	900
7-8	10	1000	9	1200
		4220		

Indirect costs are Rs. 50 per week.

**[Ans.:** (b) Normal Project Duration is 32 weeks with cost of Rs. 5820; (d) Optimum Project Duration is 29 weeks with cost of Rs. 5805]

**Question 34:** A small project is having seven activities. The relevant data about these activities is given below:

Activity	Dependence	Normal duration (Days)	Crash duration (Days)	Normal cost (Rs.)	Crash Cost (Rs.)
A	-	7	5	500	900
B	A	4	2	400	60000
C	A	5	5	500	500
D	A	6	4	800	1000
E	B,C	7	4	700	1000
F	C,D	5	2	800	1400
G	E,F	6	4	800	1600

- (i) Find out the normal duration and the minimum duration.  
 (ii) What is the percentage increase in cost to complete the project in 21 days? (10 Marks) Nov./97

**[Ans.:** (i) Normal duration 25 days, minimum duration 18 days; (ii) 15.5%]

**Miscellaneous Topics:**

**Question 35:** Write short notes on resource smoothing and resource leveling.

(5 Marks each) May/99, May/00, May/02, Nov./02, May/05

*Resource smoothing* is a resource scheduling technique used for smoothing peak resource requirement during different periods of project network. Under this technique, the constraint may be the total project duration. It helps to estimate the resource requirements for various projects. In resource smoothing, time scaled diagram of various activities of project and their floats along with their resource requirements are used. Floats on non critical activities are utilized & these activities are rescheduled or shifted (while the project duration remains unchanged) so that a uniform demand on resources is achieved.

*Resource Levelling (a.k.a. resource allocation)* is an operation of resource scheduling wherein constraint may be availability of certain resources. Here project time is varied for maximum utilization of resources i.e. project duration is not treated as an invariant, but the demand on certain specified resources should not go beyond a specified level. The maximum demand of a resource should not exceed the available limit at any point of time. Non critical activities are rescheduled by utilizing their floats.

Points which are worth noting for developing the algorithm for resource allocation:

- Halt when both resources and activities are available.
- Prior to allocation at a halt, up date the E.S.T., E.F.T. and float of the activities not allocated at earlier halt time and their succeeding activities. The repercussions may have to be traced right up to the last event.
- (a) priorities are assigned on the basis of floats e.g. 1<sup>st</sup> priority to activity with least float, 2<sup>nd</sup> to the activity with the next higher float and so on.  
 (b) In case of tie in floats, assign priorities on the basis of man-days if the activities e.g., 1<sup>st</sup> priority to the activity with highest  $M \times D$ .  
 (c) In case of tie in man-days even, assign 1<sup>st</sup> priority to the activity with highest M (gang size).  
 (d) In case of tie in M's even, assign 1<sup>st</sup> priority to the activity with lower i, where i is the tail event number of the activity.
- When an activity requires more than one man, it may so happen during allocation that the activity requires more number of persons than that available at the halt time under consideration. In such cases, the resources are allocated to the job with next priority for which they are sufficient.
- During the floating out of activities, the float of an activity may go negative which means that the project duration is going to be extended beyond the critical path. Once the float of an activity becomes negative, there from the float criterion for ascertaining priorities is invalidated. The priorities are then fixed on the basis of  $M \times D$ , gang size and lower i criteria respectively.

**Question 36:** The Madras Construction Company is bidding on a contract to install a line of microwave towers. It has identified the following activities, along with their expected time, predecessor restrictions, and worker requirements:

Activity	Duration, Weeks	Predecessor	Crew size, workers
A	4	None	4
B	7	None	2
C	3	A	2
D	3	A	4
E	2	B	3
F	2	B	3
G	2	D,E	3
H	3	F,G	4

The contract specifies that the project must be completed in 14 weeks. This company will assign a fixed number of workers to the project for its entire duration, and so it would like to ensure that the minimum number of workers is assigned and that the project will be completed in 14 weeks. Find a schedule which will do this.

[Hint.: The maximum number of workers to be assigned to the project is 6] (10 Marks) May/95

**Question 37 (Resource Smoothing):** A network with the following activity durations and manpower requirement is given. Analyze the project from point of view of resources to bring out the necessary steps involved in the analysis and smoothing of resources.

Activity:	1-2	2-3	2-4	3-5	4-6	4-7	5-8	6-8	7-9	8-10	9-10
Duration (weeks):	2	3	4	2	4	3	6	6	5	4	4
No. of Men required:	4	3	3	5	3	4	3	6	2	2	9

[Ans.: Demand for men will decrease to 15 to 11]

**Question 38 (Resource Smoothing):** Consider a project consisting of 14 activities having the duration and resource requirement shown below. Analyze the project and smoothen the requirement of the resources.

Activity	Duration (Weeks)	Masons (M)	Labourers (L)
1-2	2	1	2
2-3	3	2	2
2-4	4	3	2
2-5	2	1	3
3-10	4	2	2
4-6	2	3	2
4-7	4	3	3

5-9	4	5	3
6-8	2	1	2
7-9	5	1	3
8-9	3	-	4
9-11	2	1	1
10-11	3	1	2
11-12	2	1	2

[Ans.: Demand for masons will decrease to 8 & 10 for Labourers]

**Question 39 (Resource Allocation):** The following information is available:

Activity		No. of days	No. of men reqd. per day
A	1-2	4	2
B	1-3	2	3
C	1-4	8	5
D	2-6	6	3
E	3-5	4	2
F	5-6	1	3
G	4-6	1	8

- Draw the network and find the critical path.
- What is the peak requirement of Manpower? On which day(s) will this occur?
- If the maximum labour available on any day is only 10, when can the project be completed?

[Ans.: (i) Critical Path is AD = 10 days (ii) Peak requirement is 11 men, required on days 7 and 9 (iii) the project can be completed in 11 days.] (9 Marks) May/08

**Question 40 (Resource Allocation):** For a project consisting of several activities, the durations and required resources for carrying out each of the activities and their availabilities are given below:

- Draw the network, identify critical path and compute the total float for each of the activities.
- Find the project completion time under the given resource constraints.

Activity	Resources required		Duration (Days)
	Equipment	Operators	
1-2	X	30	4
1-3	Y	20	3
1-4	Z	20	6
2-4	X	30	4
2-5	Z	20	8
3-4	Y	20	4
3-5	Y	20	4
4-5	X	30	6

Resource availability:

No. of operators = 50, equipment X = 1, equipment Y = 1, equipment Z = 1

[Ans.: Critical Path is 1-2-4-5 with duration of 14 days, Project requires 21 days for completion under given constraints] (Nov./85)

Questions on Resource Smoothing and Resource Levelling are rarely being asked in Examination.

**Question 41 (Updating):** A company had planned its operations as follows:

Activity:	1-2	2-4	1-3	3-4	1-4	2-5	4-7	3-6	5-7	6-8	7-8
Duration (Days):	7	8	8	6	6	16	19	24	9	7	8

- Draw the network and find the critical paths.
- After 15 days of working, the following progress is noted:
  - Activities 1-2, 1-3 and 1-4 completed as per original schedule.
  - Activity 2-4 is in progress and will be completed in 4 more days.
  - Activity 3-6 is in progress and will need 17 more days to complete.
  - The staff at activity 3-6 is specialized. They are directed to complete 3-6 and undertake an activity 6-7, which will require 7 days. This rearrangement arose due to a modification in a specialization.
  - Activity 6-8 will be completed in 4 days instead of the originally planned 7 days.

(f) There is no change in the other activities.

Update the network diagram after 15 days of start of work based on the assumption given above. Indicate the revised critical paths alongwith their duration. (11 Marks) May/07

[Ans.: Critical Path 1→2→4→7→8 = 42 days; (ii) Critical path 1→3→6→7→8 = 47 days]

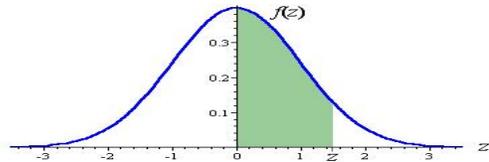
**Question 42 (Network simulation-Very Imp. for NEW course-Do it with Simulation):** A project consists of 7 activities. The time for performance of each of the activity is as follows:-

Activity	Immediate	Time	Probability
A	-	3	0.2
		4	0.6
		5	0.2
B	-	4	1.0
C	A	1	1.0
D	B,C	4	0.8
		5	0.2
E	D	3	0.1
		4	0.3
		5	0.3
		6	0.3
F	D	5	0.20
		7	0.80
G	E,F	2	0.5
		3	0.5

- Draw a network and identify critical path using expected time.
- Simulate the project for 5 times using random number and find the critical paths?

68	13	09	20	73	07	72
99	93	18	24	22	07	29
57	33	49	65	92	98	00
57	12	31	96	85	92	91
77	37	34	11	27	10	59

Table for Areas Under the Standard Normal Curve from 0 to Z (Type II)  
 $[P(0 \leq X \leq x) = n(0 \leq Z \leq z)]$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999